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An alternate Approach to Compute the Reliability of a Network with Imperfect Nodes using Binary Decision Diagrams

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ABSTRACT

In this paper i have adopted an efficient approach for calculating the reliability of the computer communication network with the help of binary decision diagram. The computer communication network has imperfect nodes and imperfect links. It means the nodes as well as the communication links may fail with known probability. The network is represented in the form of a directed graph. First i have found the reliability of the given network by applying the inclusion-exclusion formula, which is an exact method to find the reliability of a network and then i have found the reliability of the given network with binary decision diagram. To find the reliability via binary decision diagram, we must order the nodes and the communication links of the given network by applying a proposed heuristic approach. I have found that the reliability obtained via binary decision diagram is equal to the reliability obtained by inclusion-exclusion formula. I have also shown that the generated binary decision diagram is of the minimum size.

Key words: Binary Decision Diagrams (BDD), Directed Acyclic Graph (DAG), Computer communication Network (CNN), Modified Binary Decision Diagram (MBDD), Ordered Binary Decision Diagram (OBDD).

INTRODUCTION

Network reliability analysis receives considerable attention for the design, validation, and maintenance of many real world systems, such as computer, communication, or power networks. The components of a network are subject to random failures, as more and more enterprises become dependent upon computer communication network (CCN) or networked computing applications. Failure of a single component may directly affect the functioning of a network. So the probability of each component of a CCN is a crucial consideration while considering the reliability of a network. Hence the reliability consideration is an important factor in CCN[1]. The IEEE 90 standard defines the reliability as "*the ability of a system or component to perform its required functions under stated conditions for a specified period of time.*" Many algorithms have been presented to solve the problem of network reliability. These algorithms are based on the exact methods as well as the approximation methods[2-4]. Some of them are based on the min-paths/min-cuts methods. In these methods we first enumerate all the min-paths and min-cuts of the given CCN, then these min-paths/min-cuts are manipulated to get their counterpart in the sum of disjoint product form. Min-cuts methods have been used since 1960 to compute the reliability of a network[5]. The authors[6] have shown that min-cuts based algorithms are more efficient then the min-paths based algorithms only for the networks where number of min-cuts are less than the number of min-paths. However the number of min-paths/min-cuts increases exponentially as the size of the networks increases. It is impractical to enumerate all the min-paths/min-cuts of a very large network. Some of the others are based on the factoring theorem[7, 8]. Moskowitz was the first to use the factoring theorem directly to compute the reliability of a network[9]. The factoring theorem divides the reliability problems into two sub problems and the formula is given below:

$$R(G) = P_e \,.\, R(G/\text{ edge e functions}) + (1 - p_e) \,.\, R(G/\text{ edge e fail}) \quad ...(1)$$

This factoring formula must be applied only when there is no reduction on the graph is possible. It has shown that the optimal binary structure of the factoring algorithm for undirected networks can be generated by means of pivoting. Before applying the factoring, we must apply the reduction techniques like polygon to chain or series-parallel[10]. If a network has imperfect nodes as well as imperfect links, then such type of structure will increase the complexity to compute the reliability of the network. The most commonly used method for nodes failure is incident edge substitution[11]. In this method, for an edge e_i, we put $v_i e_i v_j$ in the min-path function for the perfect nodes, because if we consider an edge e_i, then this edge must contains two end vertices say v_i and v_j. So we have to simplify the Boolean function. The min-path function for the perfect nodes is the union of all min-paths from source to sink. By performing such type of operations we need large memory. One more feasible solution is to slightly change the probability function used in the factoring theorem and factor on links that have at least one end point[12]. The authors[13] have shown an efficient and exact method to compute the reliability of a network with imperfect vertices. One other method was shown by Xing with imperfect coverage and common cause failure[14].

One other author has tried to convert an undirected network in to a directed network and then compute the reliability of a network. This algorithm generates result with minor errors within reasonable time. This algorithm also generates bad result for some networks. This has been shown by Y. Chen[15]. One of the others algorithm is the brute-force algorithm. It uses the path function and have presented by V. A. Netes[16].

The network model is a directed stochastic graph G = (V, E), where V is the vertex set, and E is the set of directed edges. An incidence relation which associates with each edge of G a pair of nodes of G, called its end vertices. The edges and nodes are the components of a network that can fail with known probability. In real problems, these probabilities are usually computed from statistical data. The reliability of a network is the probability that at least one path is operational from source to sink.

This paper is organized as follows. First, we give a brief introduction to Binary decision diagrams (BDD) in section II. Then, in section III we have define three types of network reliability. In section IV, we have found the network reliability by inclusion-exclusion formula. In section V, we have proposed the description of our method for computing network reliability by using BDD. Finally we draw some conclusions in section VI.

Binary Decision Diagram

Akers[17] first introduced BDD to represent Boolean functions i.e. a BDD is a data structure used to represent a Boolean Function. Bryant[18] popularized the use of BDD by introducing a set of algorithms for efficient construction and manipulation of BDD structure. The BDD structure provides compact representations of Boolean expressions. A BDD is a directed acyclic graph (DAG) based on the Shannon decomposition. The Shannon decomposition for a Boolean function is defined as follows:

$$f = x.\, f_{x=1} + \bar{x}.\, f_{x} = 0 \quad ...(2)$$

where x is one of the decision variables, and f is the Boolean function evaluated at x = i.

By using Shannon's decomposition, any Boolean expression can be transformed in to binary tree. The authors[19] have shown a method to minimize Boolean expression with sum of disjoint product functions by using BDD. BDD are used to work out the terminal reliability of the links. In the network reliability framework, Sekine & Imai[20] have shown how to functionally construct the corresponding BDD. The authors[21] have shown an alternate approach to find the network reliability by using BDD. Table 1 shows the truth table of a Boolean function f and its corresponding Shannon tree is shown in figure 2.

Table 1: Truth Table of a Boolean Function f

X_1	X_2	X_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Sink nodes are labelled either with 0, or with 1, representing the two corresponding constant expressions. Each internal node u is labelled with a Boolean variable var(u), and has two out-edges called 0-edge, and 1-edge. The node linked by the 1-edge represents the Boolean expression when $x_i = 1$, i.e. $f_{xi=1}$; while the node linked by the 0-edge represents the Boolean expression when xi = 0, i.e. $f_{xi=0}$. The two outgoing edges are given by two functions low(u) and high(u). Indeed, such representation is space consuming. It is possible to shrink by using following three postulates.

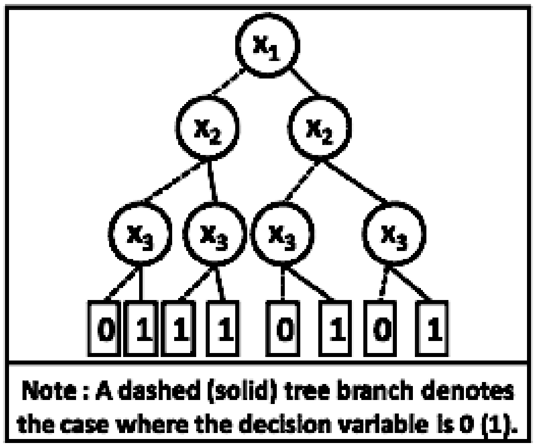

Fig. 1: Shannon Tree of the given Boolean Function

Remove Duplicate Terminals:

Delete all but one terminal vertex with a given label, and redirect all arcs into the deleted vertices to the remaining one.

Delete Redundant Non Terminals:

If non terminal vertices u, and v have var(u) = var(v), low(u) = low(v), and high(u) = high(v), then delete one of the two vertices, and redirect all incoming arcs to the other vertex.

Delete Duplicate tests :

If non terminal vertex v has low(v) = high(v), then delete v, and redirect all incoming arcs to low(v).

If we apply all these three rules then the decision tree can be reduced. The shrinking process is shown in figure 2.

Ordered Binary Decision Diagram

For an ordered BDD (OBDD), we impose a total ordering < over the set of variables and require that for any vertex u, and either non terminal child v, their respective variables must be ordered. The authors[22-23] have shown two different methods to find the reliability of the network by using OBDD.

Dual Binary Decision Diagram

If two or more BDD have the same size and representing the same Boolean function, then these BDD are known as Dual BDD, because they are Dual of each other. The size of the BDD means the total number of non terminal vertices and the number of non terminal vertices at particular level[24]. A particular sequence of variables is known as a variable ordering. It has been observed that the size of the BDD strongly depends on the ordering of variables[25]. There are three types of variable

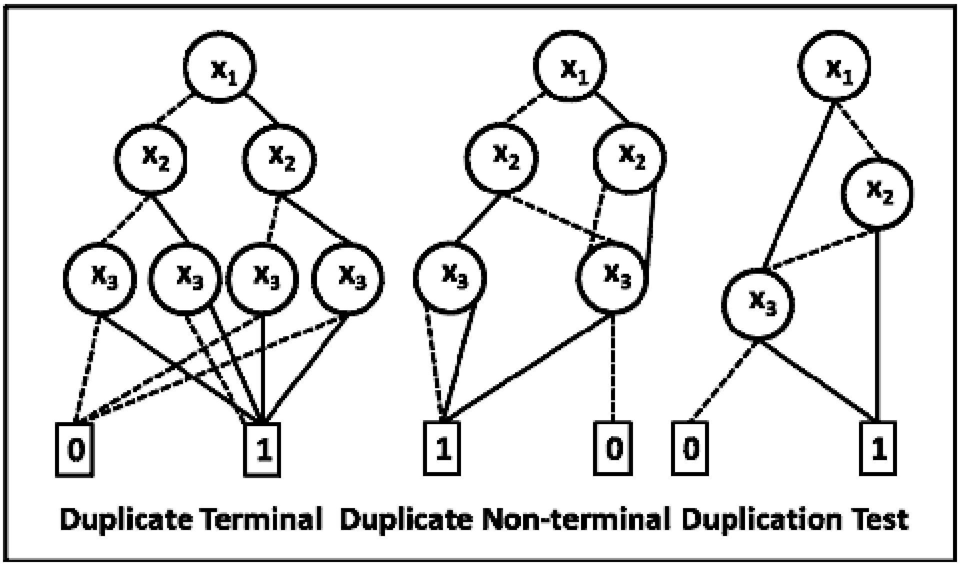

Fig. 2: Shrinking Process of the Decision Tree

ordering (optimal, good and bad) depending on the size of the different BDD [26]. An ordering is said to be optimal if it generates the minimum size BDD. A new approach for finding various optimal variable ordering to generate minimum size BDD has shown by Singhal [27]. Herrmann has shown the process how to improve the reliability of a network by using augmented BDD [28, 29].

Modified Binary Decision Diagram

The modified binary decision diagram (MBDD) is a binary decision diagram which is either dual BDD or the smaller size BDD. [30].

Network Reliability

The reliability of a network G is the probability that G supports a given operation. We distinguish three kinds of operation and hence three kind of reliability[31].

Two Terminal Reliability

It is the probability that two given vertices, called the source and the sink, can communicate. It is also called the terminal-pair reliability[32].

K Terminal reliability

When the operation requires only a few vertices, a subset k of N(G), to communicate each other, this is K terminal reliability[33].

All Terminal Reliability

When the operation requires that each pair of vertices is able to communicate via at least one operational path, this is all terminal reliability. We can see that 2-terminal reliability and all terminal reliability are the particular case of K-terminal reliability.

Computation of Network Reliability

Let us take an example of a directed network G (V, E) with single source (S) and single sink (T) as shown in the diagram.

The graph has eight nodes and nine edges. There are only three path exist from source to sink. These min-paths are as follows: $H_1 = \{S, e_1, a, e_2, b, e_3, c, e_4, d, e_5, T\}$, $H_2 = \{S, e_6, e, e_7, f, e_8, T\}$, $H_3 = \{S, e_1, a, e_2, b, e_9, e, e_7, f, e_8, T\}$.

Let $H_1, H_2,$——H_n be the n min-paths from source to sink in a network then the network connectivity function C can be represented as a logical OR of its min-paths.

$$C = H_1 U H_2 —— U H_i —— U H_n$$

So the point to point reliability is:

$$R_s = Pr\{C\} = Pr\{ H_1 U H_2 —— U H_i —— U H_n \} \quad ...(1)$$

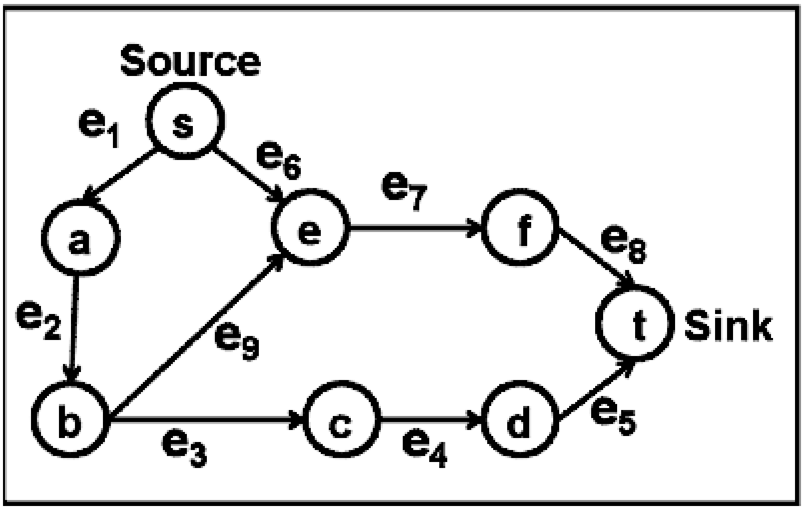

Fig. 3: A Directed Network

So the network connectivity of our network can be expressed as

$C = Se_1ae_2be_3ce_4de_5T \cup Se_6ee_7fe_8T \cup Se_1ae_2be_9ee_7fe_8T$ ___ (2)

The probability of the union of non-disjoint events, as in Formula(1), can be computed by several techniques (Exact Methods) [6]. Here we apply the inclusion-exclusion method.

Inclusion-exclusion Method

One method of transforming a Boolean expression F(G) into a probability expression is to use Poincare's theorem, also called inclusion-exclusion method [6]. The inclusion-exclusion formula for two minimal paths H_1 and H_2 is express as follows:

$E(H_1 + H_2) = E(H_1) + E(H_2) - E(H_{1.}H_2)$

Let P_i denote the probability of the node/ edge e_i of being working, by applying the Classical inclusion-exclusion formula for calculating the probability of given network (figure 3), we get

$Pr = p_Sp_1p_ap_2p_bp_3p_cp_4p_dp_5p_T + p_Sp_6p_ep_7p_fp_8p_T + p_Sp_1p_ap_2p_bp_9p_ep_7p_{Sf}p_8p_T - p_Sp_1p_ap_2p_bp_3p_cp_4p_dp_5p_6p_ep_7p_fp_8p_T - p_Sp_1p_ap_2p_bp_3p_cp_4p_dp_5p_9p_ep_7p_fp_8p_T - p_Sp_1p_2p_bp_9p_ep_7p_fp_8p_6p_T + p_Sp_1p_ap_2p_bp_3p_cp_4p_dp_5p_6p_ep_7p_fp_8p_9p_T$

Generation of BDD:

A particular sequence of variable ordering is known as variable ordering. There are three types of variable ordering namely optimal, good and bad ordering. It has been observed that the size of the BDD strongly depends on the ordering of variables. An ordering is said to be optimal if it generates the minimum size BDD [18, 25, 26]. The size of the BDD means the total number of non-terminal vertices in the BDD and the number of vertices at particular level. Here I have found an optimal ordering to generate the BDD of the given network by applying a heuristic approach.

Heuristic Approach

The heuristic approach is given below:

- Traverse the graph from source S to sink T. Find all the min-paths from source to sink.
- Check whether these paths are disjoint or not. If all the paths are disjoint then we can select any one disjoint path then second then third and so on.
- If all min-paths are not disjoint then find the out degree of the source. Since the out degree of nodes b is greater than the out degree of node e, so we can give preference to the path H_1 then middle edge e_9 and then min-path H_2.

By applying our heuristic we have found the variable ordering

$S < e_1 < a < e_2 < b < e_3 < c < e_4 < d < e_5 < e_9 < e_6 < e < e_7 < f < e_8 < T$

The BDD of the given network and probability computation is shown in figure 4.

The computation of the probability of the BDD can be calculated recursively by resorting to the Shannon decomposition.

$$Pr\{F\}= p_1 Pr\{Fx_1 =1\}+(1 - p1)Pr\{Fx_1 =0\}$$
$$= Pr\{Fx_1 =0\}+ p_1(Pr\{Fx_1 =1\}-Pr\{Fx_1 =0\})$$

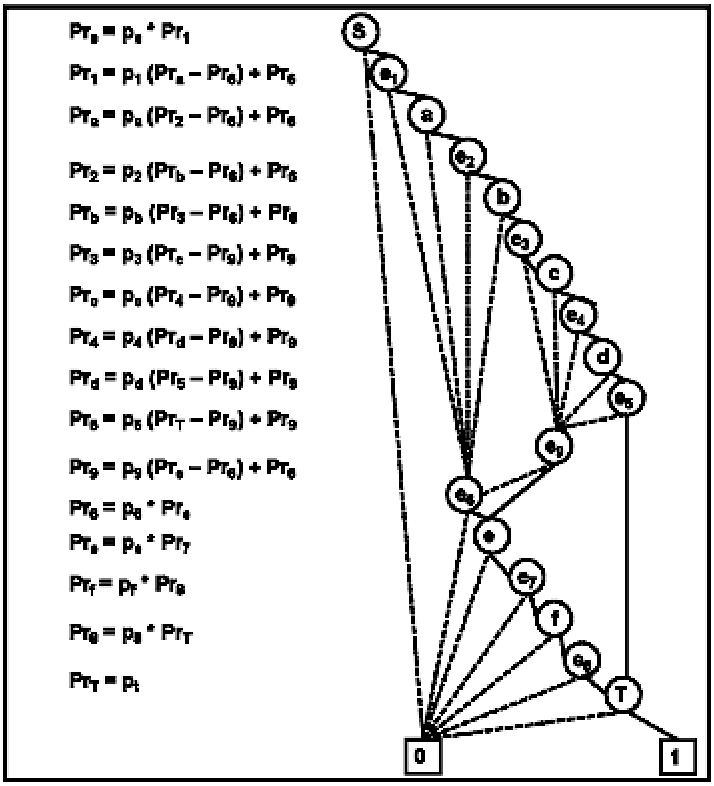

Fig. 4 BDD and its Probability Computation

where p_1 is the probability of the Boolean variable x_1 to be true and (1" p_1) is the probability of the Boolean variable x_1 to be false.

We have found that $Pr_s = p_S p_1 p_a p_2 p_b p_3 p_c p_4 p_d p_5 p_T + p_S p_6 p_e p_7 p_f p_8 p_T + p_S p_1 p_a p_2 p_b p_9 p_e p_7 p_{Sf} p_8 p_T - p_S p_1 p_a p_2 p_b p_3 p_c p_4 p_d p_5 p_6 p_e p_7 p_f p_8 p_T - p_S p_1 p_a p_2 p_b p_3 p_c p_4 p_d p_5 p_9 p_e p_7 p_f p_8 p_T - p_S p_1 p_2 p_b p_9 p_e p_7 p_f p_8 p_6 p_T + p_S p_1 p_a p_2 p_b p_3 p_c p_4 p_d p_5 p_6 p_e p_7 p_f p_8 p_9 p_T = Pr$

It has already shown that if a CCN has m disjoint min-paths then m! optimal variable ordering exist to generate the minimum size BDD[34].

Here we found that the reliability obtained by BDD is equal to the reliability obtained by inclusion-exclusion formula. There are several variables ordering are possible for constructing the different BDDs of the given CCN (figure 3) but the size is minimum only in one case. We have constructed only optimal BDD of the given CCN and compute the reliability of the given CCN by using the BDD.

CONCLUSIONS

An alternate and efficient method for generating the binary decision diagram of a computer communication network with imperfect nodes has been proposed in this paper.. I have evaluated the reliability of the given CCN via inclusion-exclusion formula and via BDD. I have found that the results (reliability) are same by both the methods. I have also found that the size of generated BDD is minimal. Our future work will focus on computing other kinds of reliability and reusing the BDD structure in order to optimize design of network topology.

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