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Prime Cordial Labeling in Context of Ringsum of Graphs

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Abstract

A bijection f from the vertex set V of a graph G to {1,2, …, |V|} is called a prime cordial labeling of G if each edge uv is assigned the label 1 if gcd (f(u), f(v)) = 1 and 0 if gcd (f(u), f(v)) > 1, where the number of edges labeled with 0 and the number of edges labeled with 1 differ at most by one. In this paper I have proved four new results admitting Prime cordial labeling.

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Keywords

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Introduction

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by V (G) and E(G) respectively. For various graph theoretic notations and terminology we follow Gross and Yellen.[3] A dynamic survey of graph labeling is published and updated every year by Gallian.[2] The concept of Sundaram *et al*[4] introduced the concept of prime cordial labeling.

Definition 1

Ring sum G_1, G_2 of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1, G_2 = (V_1, V_2, (E_1\ E_2) - (E_1 \cap E_2))$.

Main Results

Theorem 1

The graph Cn, $K_{1,n}$ is Prime cordial graph.

Proof. Let $V(G) = V_1uV_2$, where V1 = $\{u_1, u_2,..., u_n\}$ be the vertex set of C_n and $V_2 = \{v = u_1, v_1, v_2,..., vn\}$be the vertex set of $K_{1,n}$, where $v_1, v_2,..., v_n$ are pendant vertices. Note that $|V(G)| = |E(G)| = 2n$.

Define labeling function $f : V \rightarrow \{1,2,\ldots.,2n\}$ as follows.

For all $1 \le i \le n$.
$f(u_i) = 2i$,
$f(v_i) = 2i - 1$.

Then in we have $e_f(0) = e_f(1) = n$.
Therefore $|e_f(0)-e_f(1)| \le 1$
Hence C_n, $K_{1,n}$ is a Prime cordial graph.

Example 1. Prime cordial labeling of the graph C_9, $K_{1,9}$ is shown in Figure 1.

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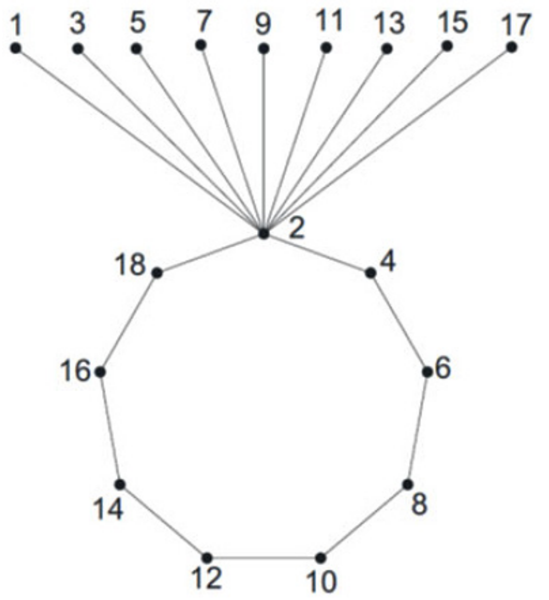

Figure. 1

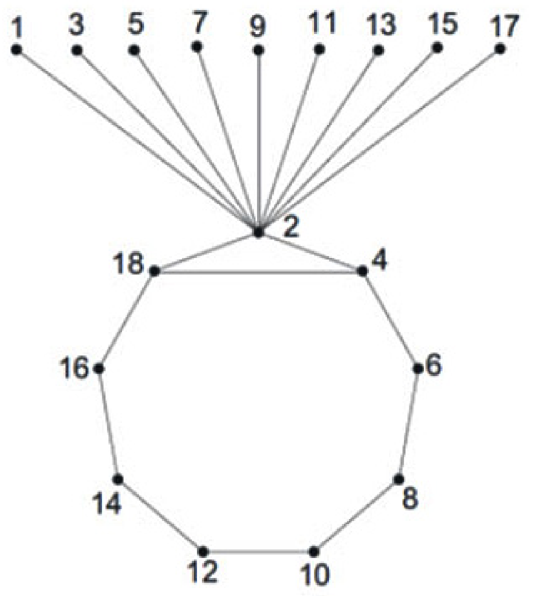

Figure. 2

Theorem 2

The graph G, $K_{1,n}$ is Prime cordial, where G is cycle C_n having one chord, where chord forms a triangle with two edges of the cycle.

Proof. Let G be the cycle C_n with one chord and Let $e = u_2u_n$ be the chord in G.

Let $V = V_1$ u V_2, where $V_1 = \{u_1, u_2,..., u_n\}$ be the vertex set of C_n and $V_2 = \{v, v_1, v_2,..., v_n\}$ be the vertex set of $K_{1,n}$. Here v is the apex vertex and v_1, v_2,..., vn are pendant vertices. Note that $v = u_1$. $|V(G)| = 2n$, $|E(G)| = 2n + 1$.

Define labeling function $f : V \rightarrow \{1,2,\ldots, 2n\}$ as follows. For all $1 \leq i \leq n$.

$f(u_i) = 2i$,

$f(v_i) = 2i - 1$.

Therefore $|e_f(0)-e_f(1)| \leq 1$

Hence G, $K_{1,n}$ is Prime cordial, where G is cycle C_n having one chord.

Example 2. Prime cordial labeling of ring sum of the graph cycle C_9 with one chord and $K_{1,9}$ is shown in Figure 2.

Theorem 3

The graph G, $K_{1,n}$ is Prime cordial for all n, where G is cycle having twin chords $C_{n,3}$.

Proof. Let G be the cycle having twin chords $C_{n,3}$, $e = u_2 u_n$ and $e' = u_3 u_n$ be the chords in G. Let $V = V_1$ u V_2, where $V_1 = \{u_1, u_2,..., u_n\}$ be the vertex set of G and $V2 = \{v, v_1, v_2,..., v_n\}$ be the vertex set of $K_{1,n}$. Here v is the apex vertex and v_1, v_2,..., v_n are pendant vertices. Note that $v = u_1$. Also, $|V(G, K_{1,n})| = 2n$, $|E(G, K_{1,n})| = 2n + 2$.

Define $f : V \rightarrow \{1,2,\ldots, 2n\}$ we conceive the below cases.

Case 1: n = 5

For all $1 \leq i \leq n$,

$f(u_4) = 5$,

$f(u_5) = 8$,

$f(u_i) = 2i$, $1 \leq i \leq 3$,

$f(v_5) = 10$,

$f(v_i) = 2i - 1$, $1 \leq i \leq 4$.

Therefore $e_f(0) = e_f(1) = n + 1$.

Case 2:
for all n except n = 5

$f(u_n) = 8$,
$f(u_i) = 2i, 1 \leq i \leq 3$,
$f(ui) = 2i - 7, 4 \leq i \leq n - 1$,

Assign the remaining vertices of star graph in any order.
Therefore $e_f(0) = e_f(1) = n + 1$.

Hence, G, $K_{1,n}$ is Prime cordial for all n, where G is cycle having twin chords $C_{n,3.}$

Example 3(a). Prime cordial labeling of ring sum of the graph cycle C5 with one chord and K1,5 is shown in Figure 3(a).

Example 3(b). Prime cordial labeling of ring sum of the graph cycle C7 with one chord and K1,7 is shown in Figure 3(b).

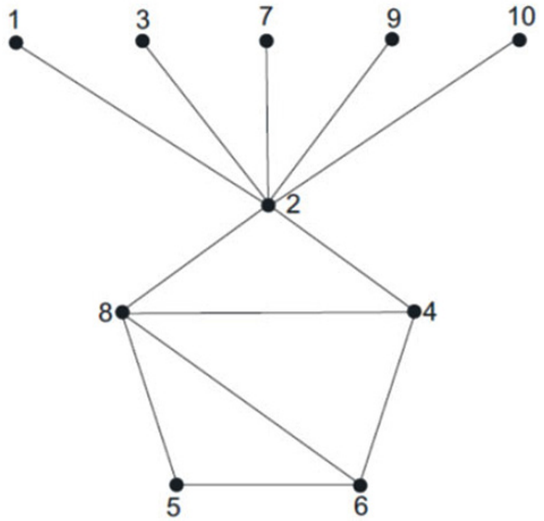

Figure. 3 (a)

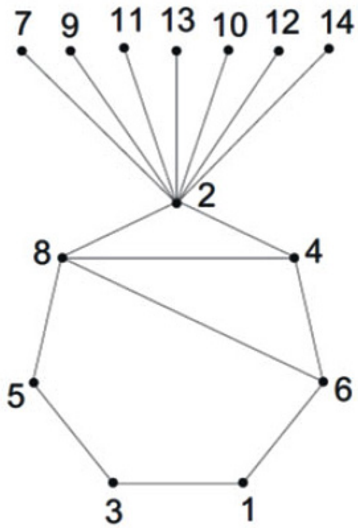

Figure. 3 (b)

Theorem 4

Pn $K_{1,n}$ is Prime cordial.

Proof. Let $V(G) = V_1 V_2$, where $V_1 = \{u_1, u_2, ..., u_n\}$ be the vertex set of P_n, $V_2 = \{v, v_1, v_2,..., v_n\}$ be the vertex set of $K_{1,n}$, where $v_1, v_2, . . . , v_n$ are pendant vertices and $v = u_1$. Note that $|V(G)| = 2n$, $|E(G)| = 2n-1$.

We define labeling function $f : V(G) \rightarrow \{F_0, F_1, F_2, ..., F_{2n}\}$, as follows.

$f(u_i) = 2i - 1, 1 \leq i \leq n$.
$f(v_i) = 2i, 1 \leq i \leq n$.

Therefore $|e_f(0) - e_f(1)| \leq 1$.
Hence, P_n , K_1,n is Prime cordial.

Example 4: Prime cordial labeling of P_5, $K_{1,5}$ is shown in figure 4.

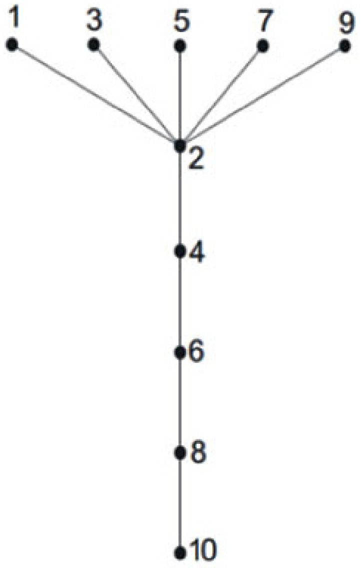

Figure. 4

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